Compositeness of gauge boson and asymptotic freedom in non-abelian gauge theory

Takashi Hattori [1]

Department of Physics, Kanagawa Dental College, Inaoka-cho, Yokosuka, Kanagawa, 238-8580, Japan

Abstract

In order to investigate the composite gauge field, we consider the compositeness condition (i.e. renormalization constant $Z_3 = 0$) in the general non-abelian gauge field theory. We calculate Z_3 at the next-to-leading order in $1/N_f$ expansion (N_f is the number of fermion flavors), and obtain the expression to the gauge coupling constant through the compositeness condition. Then the gauge coupling constant is proportional to $1/\sqrt{4N_f T(R) - 11C_2(G)}$ where $T(R)$ is the index for a representation R of gauge group G, and $C_2(G)$ is the quadratic Casimir. It is found that the gauge boson compositeness take place only when $N_f T(R)/C_2(G) > 11/4$, in which the asymptotic freedom in the non-abelian gauge field theory fails.

[1] e-mail : hattorit@kdcnet.ac.jp

1 INTRODUCTION

The compositeness of vector boson [1]–[12] is a interesting subject in present particle physics. The idea of composite gauge boson has been applied in quark-lepton physics [2] and hadron physics [3]. The experimental results [13] do not exclude the possibility of compositeness of gauge boson. In the composite gauge field theory, the bare Lagrangian does not include explicitly the kinetic term of gauge field. The kinetic term of gauge field, however, be derived from the quantum fluctuations of matter field, in which the gauge field becomes consequently dynamical one, and then the gauge boson is regarded as a composite which is composed of the matter fields. When we consider the higher-order contributions of the model, tremendous divergences prevent us to solve perturbatively the problem of composite state. In order to obtain physical predictions, people use the ladder approximation or the renormalization group equation. In these method, the contributions from particular diagrams are calculated through the expansion with gauge coupling constant. For a solution of the problem of composite state, we use the gauge theory with the compositeness condition [6], and then consider the contributions from next-to-leading diagrams through the $1/N_f$ expansion where N_f is the number of matter-fermion flavors. In general, the gauge field theory with the compositeness condition $Z_3 = 0$ is reduced to the composite gauge field theory with the finite cutoff Λ (i.e. new physics scale) where Z_3 is the wave function renormalization constant of the gauge field. The gauge field theory with $Z_3 = 0$ provides a useful method to investigate the composite gauge field when the physical cutoff scale Λ is very large than the present energy scale. Hence we calculate the renormalization constant Z_3 with the $1/N_f$ expansion within the gauge field theory. Then the information of composite gauge field theory can be obtained from the gauge field theory with $Z_3 = 0$.

In the previous papers [12], K. Akama and the present author investigated the compositeness condition in abelian ($U(1)$) and non-abelian ($SU(N_c)$) gauge field theories at next-to-leading order $1/N_f$. The compositeness condition in Nambu-Jona-Lasinio model [14] at the next-to-leading order was investigated in Ref. [15].

In this paper, we present a detailed formulation of the scheme for the compositeness condition of general non-abelian gauge field, and discuss *a complementarity* between gauge boson compositeness and asymptotic freedom [16] in the gauge field theory. In Sec. II, we present the composite gauge field which is composed of matter fermions, and then discuss the gauge field theory with compositeness condition ($Z_3 = 0$). In Sec. III, we calculate the renormalization constant Z_3 to the next-to-leading order in $1/N_f$ expansion [12] within the general non-abelian gauge field theory. In Sec. IV, we obtain the expression to the gauge coupling constant through the compositeness condition. In Sec. V, we give the conclusions of this paper.

2 COMPOSITENESS AND RENORMALIZATION

2.1 Compositeness condition

We consider the composite gauge field theory with N_f matter fermions ψ^j ($j = 1, 2, \cdots, N_f$) each of which belong to the representation of color gauge group G. The Lagrangian is given by the following gauge invariant form without the kinetic term of bare gauge field,

$$\mathcal{L}_A = \overline{\psi}^j \, (i \, \partial\!\!\!/ - m + T^a A\!\!\!/^a) \, \psi^j , \tag{1}$$

where T^a is the generator of gauge group G, and m is the bare mass of matter fermions ψ^j. For example, if we consider the matter fermions ψ^j which belong to the fundamental representation of $SU(N_c)$ group, then T^a is given by $T^a = \lambda^a/2$ $(a = 1, 2, \cdots, N_c^2 - 1)$, where λ^a are the $SU(N_c)$ Gell-Mann matrices. In (1), the fields A_μ^a may be regarded as auxiliary field without independent dynamical degree of freedom, because if we use the Euler equation with respect to A_μ^a for (1), then $\partial\mathcal{L}_A/\partial A_\mu^a = 0$ give the constraint condition with respect to A_μ^a, as follows

$$\overline{\psi}^j T^a \gamma_\mu \psi^j = 0. \tag{2}$$

The kinetic term of non-abelian gauge field is induced through quantum fluctuation of the fermion fields ψ^j, and hence the gauge field becomes dynamical one. This implies that the gauge boson is a composite made of a pair of the fermion ψ^j and its anti-particle. We note that (1) corresponds to strong coupling limit $(F \to \infty)$ of the following Nambu-Jona-Lasinio type model, where F is the coupling constant

$$\mathcal{L}_N = \overline{\psi}^j \left(i\,\partial\!\!\!/ - m\right)\psi^j + F\left(\overline{\psi}^j T^a \gamma_\mu \psi^j\right)^2 \tag{3}$$

which is equivalent to

$$\mathcal{L}'_A = \overline{\psi}^j \left(i\,\partial\!\!\!/ - m\right)\psi^j + \overline{\psi}^j T^a A\!\!\!/^a \psi^j - \frac{1}{4F}(A_\mu^a)^2. \tag{4}$$

The theory (1) involves severe ultraviolet divergences. In order to absorb the divergences in part, we rescale the fields and the mass as

$$\psi^j = \sqrt{X_\psi}\,\psi_{\mathrm{r}}^j, \qquad A_\mu^a = \sqrt{X_A}\,G_{\mathrm{r}\mu}^a, \qquad m = \frac{X_m}{X_\psi} m_{\mathrm{r}}, \tag{5}$$

then (1) is rewritten as

$$\mathcal{L}_A = X_\psi \overline{\psi}_{\mathrm{r}}^j \left(i\,\partial\!\!\!/ - \frac{X_m}{X_\psi} m_{\mathrm{r}}\right)\psi_{\mathrm{r}}^j + X_\psi \sqrt{X_A}\;\; \overline{\psi}_{\mathrm{r}}^j T^a\, G\!\!\!/_{\mathrm{r}}^a \psi_{\mathrm{r}}^j, \tag{6}$$

where X_ψ, X_A, and X_m are rescaling factors for the bare quantities ψ^j, A_μ^a, and m, respectively.

Now we see that the composite gauge field theory (1) is the special case of the ordinary gauge field theory specified by the compositeness condition $Z_3 = 0$. The non-abelian gauge theory with gauge field G_μ^a and fermionic fields ψ^j is given by the Lagrangian

$$\mathcal{L}_G = -\frac{1}{4}\left(G_{\mu\nu}^a\right)^2 + \overline{\psi}^j \left(i\,\partial\!\!\!/ - m + gT^a G\!\!\!/^a\right)\psi^j \tag{7}$$

with $G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g f^{abc} G_\mu^b G_\nu^c$, where g is the gauge coupling constant, and f^{abc} is the structure constant of gauge group G. In order to absorb the ultraviolet divergences in quantum corrections, we renormalize the bare quantities in (7) as

$$\psi^j = \sqrt{Z_2}\,\psi_{\mathrm{r}}^j, \qquad G_\mu^a = \sqrt{Z_3}\,G_{\mathrm{r}\mu}^a, \qquad g = \frac{Z_1}{Z_2\sqrt{Z_3}} g_{\mathrm{r}}, \qquad m = \frac{Z_m}{Z_2} m_{\mathrm{r}}, \tag{8}$$

then (7) is rewritten as

$$\mathcal{L}_G = -\frac{1}{4} Z_3 \left(G^a_{\mathrm{r}\mu\nu} \right)^2 + Z_2 \overline{\psi}^j_{\mathrm{r}} \left(i \partial\!\!\!/ - \frac{Z_m}{Z_2} m_{\mathrm{r}} + \frac{Z_1}{Z_2} g_{\mathrm{r}} T^a G\!\!\!/^a_{\mathrm{r}} \right) \psi^j_{\mathrm{r}} \tag{9}$$

with $G^a_{\mathrm{r}\mu\nu} = \partial_\mu G^a_{\mathrm{r}\nu} - \partial_\nu G^a_{\mathrm{r}\mu} + (Z_1/Z_2) g_{\mathrm{r}} f^{abc} G^b_{\mathrm{r}\mu} G^c_{\mathrm{r}\nu}$, where the quantities with the subscript "r" are the renormalized ones, and Z_1, Z_2, Z_3, and Z_m are the renormalization constants. When we use the following copmositeness condition for the non-abelian gauge field,

$$Z_3 = 0, \tag{10}$$

(9) reduces to

$$\mathcal{L}_G = Z_2 \overline{\psi}^j_{\mathrm{r}} \left(i \partial\!\!\!/ - \frac{Z_m}{Z_2} m_{\mathrm{r}} + \frac{Z_1}{Z_2} g_{\mathrm{r}} T^a G\!\!\!/^a_{\mathrm{r}} \right) \psi^j_{\mathrm{r}}, \tag{11}$$

in which the kinetic term of non-abelian gauge field $G^a_{\mathrm{r}\mu}$ vanishes. If we set the following equalities,

$$Z_2 = X_\psi, \qquad \frac{Z_m}{Z_2} = \frac{X_m}{X_\psi}, \qquad Z_1 g_{\mathrm{r}} = X_\psi \sqrt{X_A}, \tag{12}$$

then (11) is identical to (6). Hence the non-abelian gauge field theory (7) with $Z_3 = 0$ is equivalent to the composite gauge field theory (1).

Hereafter we consider the gauge field theory (7) with compositeness condition ($Z_3 = 0$) in order to investigate the composite gauge field. In the calculation of quantum effects, we need to fix the gauge to guarantee that the inversed gauge boson propagator exists. Then the gauge fixing term and the Faddeev-Popov ghost term are introduced into (7) and (9), as follows

$$\begin{aligned} \mathcal{L}^{\mathrm{g.f.}}_G &= \mathcal{L}_G - \frac{1}{2\xi} \left(\partial^\mu G^a_\mu \right)^2 + \partial^\mu \eta^{a\dagger} \left(\partial_\mu \eta^a - g f^{abc} \eta^b G^c_\mu \right) \\ &= \mathcal{L}_G - \frac{1}{2\xi_{\mathrm{r}}} \left(\partial^\mu G^a_{\mathrm{r}\mu} \right)^2 + Z_\eta \partial^\mu \eta^{a\dagger}_{\mathrm{r}} \left(\partial_\mu \eta^a_{\mathrm{r}} - \frac{Z_1}{Z_2} g_{\mathrm{r}} f^{abc} \eta^b_{\mathrm{r}} G^c_{\mathrm{r}\mu} \right), \end{aligned} \tag{13}$$

where ξ is the gauge fixing parameter, and η^a ($a = 1, 2, \cdots, N_c^2 - 1$) is the Faddeev-Popov ghost field, and Z_η is the renormalization constant for η^a. The gauge fixing in (13) dos not alter the physical quantities, and hence (13) is equivalent to (7) and (9) [17].

2.2 Leading order contribution to Z_3

In order to find Z_3, we use rather $1/N_f$ expansion than usual perturbation expansion with the coupling constant g_{r}, because the latter expansion fails under the compositeness condition ($Z_3 = 0$) [12]. At the leading order in $1/N_f$ expansion, Z_3 is chosen so as to cancel the divergence to the one-fermion-loop diagram A in Fig.1, where the solid and wavy lines indicate the fermion and the gauge boson propagators, respectively. In the dimensional regularization, the one-loop vacuum polarization tensor is given by

$$\Pi^{ab\ \mathrm{A}}_{\mu\nu}(p) = \frac{4}{3} \frac{1}{(4\pi)^2} N_f g_{\mathrm{r}}^2 \Gamma(\epsilon) \frac{1}{(-p^2)^\epsilon} \left(-g_{\mu\nu} p^2 + p_\mu p_\nu \right) \mathrm{Tr}(T^a T^b) + O(m_{\mathrm{r}}^2), \tag{14}$$

where p_μ is the momentum of external gauge boson line, and ϵ is given by $\epsilon = (4-d)/2$ with the number of space-time dimensions d. Then the renormalized tensor $\Pi^{ab\ \mathrm{A}}_{\mu\nu\ \mathrm{ren}}(p)$ is given by

$$\Pi^{ab\ \mathrm{A}}_{\mu\nu\ \mathrm{ren}}(p) = \Pi^{ab\ \mathrm{A}}_{\mu\nu}(p) + (Z_3 - 1)\left(-g_{\mu\nu}p^2 + p_\mu p_\nu\right)\delta^{ab}. \tag{15}$$

The leading divergent part $\Pi^{ab\ \mathrm{A}}_{\mu\nu\ \mathrm{div}}(p)$ is expressed as

$$\Pi^{ab\ \mathrm{A}}_{\mu\nu\ \mathrm{div}}(p) = \frac{4}{3}N_f T(R) g_\mathrm{r}^2 I\left(-g_{\mu\nu}p^2 + p_\mu p_\nu\right)\delta^{ab} \tag{16}$$

with $T(R)\delta^{ab} = \mathrm{Tr}(T^a T^b)$, where I is given by

$$I = \frac{1}{16\pi^2}\Gamma(\epsilon) \approx \frac{1}{16\pi^2\epsilon}. \tag{17}$$

If we use the Pauli-Villars regularization, I is given by

$$I = \frac{1}{16\pi^2}\ln\left(\frac{\Lambda^2}{m_\mathrm{r}^2}\right), \tag{18}$$

where Λ is the momentum cutoff. In order to cancel the divergence, we choose Z_3 in (15), as follows

$$Z_3 = 1 - \frac{4}{3}N_f T(R) g_\mathrm{r}^2 I. \tag{19}$$

For the renormalization constant Z_3, (19) gives the contribution at the leading order $O(1)$ in $1/N_f$ expansion.

When we use the compositeness condition (10) to (19), the gauge coupling constant g_r is expressed as

$$g_\mathrm{r}^2 = \frac{3}{4N_f T(R) I} \propto \frac{\epsilon}{N_f}. \tag{20}$$

For the gauge coupling constant g_r, (20) gives the result at the leading order $O(N_f^{-1})$ in $1/N_f$ expansion. Notice that (20) coincides with the known result of the composite gauge field theory [1]–[2].

We take the regularization scheme as an approximation to some physical momentum cutoff Λ which implies the new physics scale. Then the parameter ϵ should be fixed at the non-vanishing value from (17) and (18), as follows

$$\epsilon = \frac{1}{\ln(\Lambda^2/m_\mathrm{r}^2)}. \tag{21}$$

In (20), we notice that the gauge coupling constant g_r is proportional to $\sqrt{\epsilon/N_f}$.

3 NEXT-TO-LEADING CONTRIBUTIONS TO Z_3

3.1 Leading divergences

We consider the next-to-leading contributions to renormalization constant Z_3 in $1/N_f$ expansion. The diagrams are classified by powers of $1/N_f$ instead of powers of the coupling constant g_r. In Fig.1 (B–H),

we show the diagrams for the next-to-leading contribution in the $1/N_f$ expansion, where the disks stand for insertions of an arbitrary number of one-fermion loops into the gauge boson propagator. In addition to the one-boson-loop diagrams B and C, the multi-loop diagrams (D–H) belong to this order, because the gauge boson propagator with the fermion-loops inserted has the same order in N_f^{-1} as the usual gauge boson propagator. Though the integral of an n-loop diagram diverges like $(1/\epsilon)^n$, the diagram is suppressed by the factor $g_{\mathrm{r}}^{2n} \propto \epsilon^n$ from (20). Therefore, all the diagrams in Fig.1 behave like $O(\epsilon^0)$ at least. Hereafter we return only the $O(\epsilon^0)$ contributions, which implies that we retain only the leading divergences.

The contribution from diagram A in Fig.1 was discussed already in the previous section in which the diagram gives the result of leading order in $1/N_f$ expansion. The contribution from diagram B is given by

$$\Pi_{\mu\nu}^{ab\ \mathrm{B}} = \frac{g_{\mathrm{r}}^2}{16\pi^2}\Gamma(\epsilon)\frac{1}{(-p^2)^\epsilon}\left[\frac{25}{12}g_{\mu\nu}p^2 - \frac{7}{3}p_\mu p_\nu - \frac{1}{2}\xi_{\mathrm{r}}\left(g_{\mu\nu}p^2 - p_\mu p_\nu\right)\right] C_2(G)\delta^{ab} \tag{22}$$

with $C_2(G)\delta^{ab} = f^{acd}f^{bcd}$. The contribution from the Faddeev-Popov-ghost loop diagram C in Fig.1 is given by

$$\Pi_{\mu\nu}^{ab\ \mathrm{C}} = -\frac{g_{\mathrm{r}}^2}{16\pi^2}\Gamma(\epsilon-1)\frac{1}{(-p^2)^\epsilon}\left(\frac{1}{12}g_{\mu\nu}p^2 + \frac{1}{6}p_\mu p_\nu\right) C_2(G)\delta^{ab}. \tag{23}$$

Now we discuss the each contribution from multi-loop diagrams D-H in Fig.1. First let us present the gauge boson propagator $D_{\mu\nu}^{ab\ [l]}(p)$ into which an arbitrary number of one-fermion loops are inserted, where superscript l is the number of one-fermion loops which are inserted into the gauge boson propagator. The gauge boson propagator $D_{\mu\nu}^{ab\ [l]}(p)$ is defined as

$$D_{\mu\nu}^{ab\ [l]}(p) = D_{\mu\mu_1}^{aa_1}(p)\ \Pi_{a_1 b_1}^{\mu_1\nu_1}(p)\ D_{\nu_1\mu_2}^{b_1 a_2}(p)\ \Pi_{a_2 b_2}^{\mu_2\nu_2}(p)\ \cdots\ \Pi_{a_l b_l}^{\mu_l\nu_l}(p)\ D_{\nu_l\nu}^{b_l b}(p), \tag{24}$$

where $D_{\mu\mu_1}^{aa_1}(p)$, etc. are the usual gauge boson propagators, and $\Pi_{a_1 b_1}^{\mu_1\nu_1}(p)$, etc. are given by (14). Then we obtain the gauge boson propagator $D_{\mu\nu}^{ab\ [l]}(p)$, as follows

$$D_{\mu\nu}^{ab\ [l]}(p) = \frac{1}{(-p^2)^{1+\epsilon l}}\left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right)^l \left[-g_{\mu\nu} + (1-\xi_{\mathrm{r}}\delta_{0l})\frac{p_\mu p_\nu}{p^2}\right]\delta^{ab} \tag{25}$$

with $I = \Gamma(\epsilon)/16\pi^2 \approx 1/(16\pi^2\epsilon)$, where δ_{0l} is Kronecker's symbol. For $l = 0$, (25) is reduced to the usual gauge boson propagator $D_{\mu\nu}^{ab}(p)$.

The contribution from diagram D is given by

$$\Pi_{\mu\nu}^{ab\ \mathrm{D}}(p) = -N_f \int \frac{d^n k}{i(2\pi)^n}\mathrm{Tr}\left[\frac{1}{m_{\mathrm{r}} - \not{k}}J(k)\frac{1}{m_{\mathrm{r}} - \not{k}}\gamma_\mu\left(g_{\mathrm{r}}T^a\right)\frac{1}{m_{\mathrm{r}} - (\not{p} + \not{k})}\gamma_\nu\left(g_{\mathrm{r}}T^b\right)\right] \tag{26}$$

with the fermion self-energy part

$$J(k) = -\int \frac{dq^n}{i(2\pi)^n}\left[\gamma^\rho\left(g_{\mathrm{r}}T^c\right)\frac{1}{m_{\mathrm{r}} - (\not{k} - \not{q})}\gamma^\sigma\left(g_{\mathrm{r}}T^d\right)D_{\rho\sigma}^{cd\ [l]}(q)\right]. \tag{27}$$

Then the fermion self-energy part (27) is calculated as

$$J(k) = \frac{g_{\rm r}^2}{16\pi^2}\Gamma\left(\epsilon l+\epsilon\right)\left(-\frac{4}{3}N_f T(R) g_{\rm r}^2 I\right)^l \frac{\not{k}}{(-k^2)^{\epsilon(l+1)}}\left[-1+(1-\delta_{0l}\xi_{\rm r})\right] T^c T^d \delta^{cd} + O(m_{\rm r}^2). \tag{28}$$

When the gauge boson propagator $D_{\rho\sigma}^{cd\ [l]}(q)$ in (27) involves the fermion loop (i.e. $l \neq 0$), the term of $O(\epsilon^0)$ in (28) vanishes, and then the vacuum polarization tensor $\Pi_{\mu\nu}^{ab\ \rm D}(p)$ does not contribute to Z_3. When $D_{\rho\sigma}^{cd\ [l]}(q)$ does not involve the fermion loop (i.e. $l = 0$), (28) is reduced to

$$J(k) = -\xi_{\rm r}\frac{g_{\rm r}^2}{16\pi^2}\Gamma\left(\epsilon\right)\frac{\not{k}}{(-k^2)^{\epsilon}} T^c T^d \delta^{cd} + O(m_{\rm r}^2). \tag{29}$$

Hence we obtain the contribution of $O(\epsilon^0)$ in diagram D with $l = 0$, as follows

$$\Pi_{\mu\nu}^{ab\ \rm D}(p) = \xi_{\rm r}\frac{g_{\rm r}^2}{8\pi^2}\left(-\frac{4}{3}N_f T(R) g_{\rm r}^2 I\right)\Gamma(2\epsilon-1)\frac{1}{(-p^2)^{2\epsilon}}\left(g_{\mu\nu}p^2 - p_\mu p_\nu\right)\mathrm{Tr}\left(T^a T^b T^c T^d\right)\delta^{cd}. \tag{30}$$

The contribution from diagram E is given by

$$\Pi_{\mu\nu}^{ab\ \rm E}(p) = -N_f\int\frac{d^n k}{i(2\pi)^n}\mathrm{Tr}\left[\Lambda_\mu^a(k)\frac{1}{m_{\rm r}-(\not{p}+\not{k})}\gamma_\nu\left(g_{\rm r}T^b\right)\frac{1}{m_{\rm r}-\not{k}}\right] \tag{31}$$

with the vertex correction part

$$\Lambda_\mu^a(k) = 2\int\frac{d^n q}{i(2\pi)^n}D_{\rho\sigma}^{cd\ [l]}(q-k)\gamma^\sigma\left(g_{\rm r}T^d\right)\frac{1}{m_{\rm r}-\not{q}}\gamma_\mu\left(g_{\rm r}T^a\right)\frac{1}{m_{\rm r}-(\not{p}+\not{q})}\gamma^\rho\left(g_{\rm r}T^c\right), \tag{32}$$

where the factor 2 arise from the overlapping divergence. The vertex correction part (32) is calculated as

$$\Lambda_\mu^a(k) = \frac{2g_{\rm r}^3}{16\pi^2}\left(-\frac{4}{3}N_f T(R) g_{\rm r}^2 I\right)^l\Gamma\left(\epsilon l+\epsilon\right)\frac{\gamma_\mu}{(-k^2)^{\epsilon(l+1)}}\left[1-(1-\delta_{0l}\xi_{\rm r})\right] T^d T^a T^c \delta^{cd} + O(m_{\rm r}^2). \tag{33}$$

When the gauge boson propagator $D_{\rho\sigma}^{cd\ [l]}(q-k)$ in (32) involves the fermion loop (i.e. $l \neq 0$), the term of $O(\epsilon^0)$ in (33) vanishes, and then $\Pi_{\mu\nu}^{ab\ \rm E}(p)$ does not contribute to Z_3. When $D_{\rho\sigma}^{cd\ [l]}(q-k)$ does not involve the fermion loop (i.e. $l = 0$), (33) is reduced to

$$\Lambda_\mu^a(k) = \xi_{\rm r}\frac{2g_{\rm r}^3}{16\pi^2}\Gamma(\epsilon)\frac{\gamma_\mu}{(-k^2)^{\epsilon}} T^d T^a T^c \delta^{cd} + O(m_{\rm r}^2). \tag{34}$$

Hence we obtain the contribution of $O(\epsilon^0)$ in diagram E with $l = 0$, as follows

$$\Pi_{\mu\nu}^{ab\ \rm E}(p) = -\xi_{\rm r}\frac{2g_{\rm r}^2}{8\pi^2}\left(-\frac{4}{3}N_f T(R) g_{\rm r}^2 I\right)\Gamma(2\epsilon-1)\frac{1}{(-p^2)^{2\epsilon}}\left(g_{\mu\nu}p^2 - p_\mu p_\nu\right)\mathrm{Tr}\left(T^d T^a T^c T^b\right)\delta^{cd}. \tag{35}$$

The contribution from diagram F is given by

$$\Pi_{\mu\nu}^{ab\ \rm F}(p) = -\frac{1}{2}\int\frac{d^n q}{i(2\pi)^n}(ig_{\rm r}f^{bcc'})\Xi^{\nu\beta\beta'}(q,p)D_{\alpha\beta}^{cd\ [l]}(q)D_{\alpha'\beta'}^{c'd'\ [l']}(p-q)(ig_{\rm r}f^{add'})\Xi^{\mu\alpha\alpha'}(q,p) \tag{36}$$

with

$$\Xi^{\nu\beta\beta'}(q,p) = g^{\beta\beta'}(p-2q)^{\nu} + g^{\beta'\nu}(q-2p)^{\beta} + g^{\nu\beta}(p+q)^{\beta'}$$
$$\Xi^{\mu\alpha\alpha'}(q,p) = g^{\alpha\alpha'}(p-2q)^{\mu} + g^{\alpha'\mu}(q-2p)^{\alpha} + g^{\mu\alpha}(p+q)^{\alpha'}, \tag{37}$$

where l and l' are the numbers of one-fermion loops which are inserted into the each gauge boson propagator in Fig.1 (F–H). Then we obtain the contribution of $O(\epsilon^0)$ from diagram F,

$$\begin{aligned}\Pi^{ab\ \mathrm{F}}_{\mu\nu}(p) &= \frac{g_{\mathrm{r}}^2}{16\pi^2}\left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right)^{l+l'} \Gamma\left(\epsilon l + \epsilon l' + \epsilon\right) \\ &\times \frac{1}{(-p^2)^{\epsilon(l+l'+1)}}\left[\frac{25}{12}g_{\mu\nu}p^2 - \frac{7}{3}p_\mu p_\nu - \frac{1}{4}\xi_{\mathrm{r}}(\delta_{0l}+\delta_{0l'})\left(g_{\mu\nu}p^2 - p_\mu p_\nu\right)\right] C_2(G)\delta^{ab}. \end{aligned}\tag{38}$$

In (36), when the gauge boson propagators $D^{cd\ [l]}_{\alpha\beta}(q)$ and $D^{c'd'\ [l']}_{\alpha'\beta'}(p-q)$ does not involve the fermion loop (i.e. $l = l' = 0$), then (38) is equal to (22) which is the contribution of the diagram B. Hereafter we suppress the mass of fermion in the propagators because the term involving m_{r} in Z_3 behave like $O(\epsilon)$ or less, while we are retaining $O(\epsilon^0)$ terms, which are leading in ϵ (see the first paragraph in this section).

The contribution from diagram G is given by

$$\Pi^{ab\ \mathrm{G}}_{\mu\nu}(p) = -\frac{1}{2}\int\frac{d^nq}{i(2\pi)^n}\int\frac{d^nk}{i(2\pi)^n}(ig_{\mathrm{r}}f^{bcc'})\Xi^{\nu\beta\beta'}(q,p)D^{cd\ [l]}_{\alpha\beta}(q)D^{c'd'\ [l']}_{\alpha'\beta'}(p-q)\mathcal{F}^{add'}_{\mu\alpha\alpha'}(q,k,p) \tag{39}$$

with the three boson vertex part

$$\begin{aligned}\mathcal{F}^{add'}_{\mu\alpha\alpha'}(q,k,p) &= -N_f \mathrm{Tr}\left[\frac{1}{-(\not{k}+\not{p})}(g_{\mathrm{r}}T^a)\gamma_\mu\frac{1}{-\not{k}}(g_{\mathrm{r}}T^d)\gamma_\alpha\frac{1}{-(\not{k}+\not{q})}(g_{\mathrm{r}}T^{d'})\gamma_{\alpha'}\right] \\ &\quad -N_f \mathrm{Tr}\left[\frac{1}{-(\not{k}-\not{q})}(g_{\mathrm{r}}T^d)\gamma_\alpha\frac{1}{-\not{k}}(g_{\mathrm{r}}T^a)\gamma^\mu\frac{1}{-(\not{k}-\not{p})}(g_{\mathrm{r}}T^{d'})\gamma_{\alpha'}\right].\end{aligned}\tag{40}$$

We note that in (40) the fermion propagators $-(\not{k}+\not{p})^{-1}$ and $-(\not{k}-\not{p})^{-1}$ are expressed as

$$\frac{1}{-(\not{k}+\not{p})} = \frac{1}{-\not{k}} + \frac{1}{-\not{k}}\ \not{p}\frac{1}{-\not{k}} + \frac{1}{-\not{k}}\ \not{p}\frac{1}{-\not{k}}\ \not{p}\frac{1}{-(\not{k}+\not{p})} \tag{41}$$

and

$$\frac{1}{-(\not{k}-\not{p})} = \frac{1}{-\not{k}} + \frac{1}{-\not{k}}(-\not{p})\frac{1}{-\not{k}} + \frac{1}{-\not{k}}(-\not{p})\frac{1}{-\not{k}}(-\not{p})\frac{1}{-(\not{k}-\not{p})}, \tag{42}$$

respectively. For overlapping divergence in the diagram G, we separate two parts as diagrams $\mathrm{G_f}$ and $\mathrm{G_m}$ (Fig.2). The subscript "f" indicates the contribution where the divergence occurs at the fermion loop subdiagram which is inserted to the three gauge boson vertex part (Fig.2 $\mathrm{G_f}$), while the subscript "m" indicates that where the divergence occurs at the boson-fermion-boson (mixed) loop subdiagram in the boson-fermion-fermion vertex part (Fig.2 $\mathrm{G_m}$). When we retain only the leading divergence, the first and second terms in (41) and (42) contribute to the diagram $\mathrm{G_f}$, and the third term contributes to the diagram $\mathrm{G_m}$. The contribution from diagram $\mathrm{G_f}$ is given by

$$\Pi^{ab\ \mathrm{G_f}}_{\mu\nu}(p) = -\frac{1}{2}\int\frac{d^nq}{i(2\pi)^n}(ig_{\mathrm{r}}f^{bcc'})\Xi^{\nu\beta\beta'}(q,p)D^{cd\ [l]}_{\alpha\beta}(q)D^{c'd'\ [l']}_{\alpha'\beta'}(p-q)F^{add'}_{\mu\alpha\alpha'}(q,p) \tag{43}$$

with the three gauge boson vertex part

$$
\begin{aligned}
&F^{add'}_{\mu\alpha\alpha'}(q,p) \\
&= -g_{\mathrm{r}}^3 N_f \int \frac{d^n k}{i(2\pi)^n}\Bigg\{ \mathrm{Tr}(T^a T^d T^{d'})\mathrm{Tr}\left[\left(\frac{1}{-\not k} + \frac{1}{-\not k}\not p\frac{1}{-\not k}\right)\gamma_\mu \frac{1}{-\not k}\gamma_\alpha \frac{1}{-(\not k + \not q)}\gamma_{\alpha'}\right] \\
&\qquad\qquad + \mathrm{Tr}(T^d T^a T^{d'})\mathrm{Tr}\left[\frac{1}{-(\not k - \not q)}\gamma_\alpha \frac{1}{-\not k}\gamma_\mu \left(\frac{1}{-\not k} + \frac{1}{-\not k}(-\not p)\frac{1}{-\not k}\right)\gamma_{\alpha'}\right]\Bigg\}. \qquad (44)
\end{aligned}
$$

Then the three gauge boson vertex part $F^{add'}_{\mu\alpha\alpha'}(q,p)$ is calculated as

$$
F^{add'}_{\mu\alpha\alpha'}(q,p) = ig_{\mathrm{r}} f^{add'}\left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right)\frac{1}{(-q^2)^\epsilon}\left[g_{\mu\alpha}(p+q)_{\alpha'} + g_{\mu\alpha'}(q-2p)_\alpha + g_{\alpha\alpha'}(p-2q)_\mu\right]. \qquad (45)
$$

Hence we obtain the contribution of $O(\epsilon^0)$ in diagram $\mathrm{G_f}$, as follows

$$
\begin{aligned}
\Pi^{ab\ \mathrm{G_f}}_{\mu\nu}(p) &= -\frac{g_{\mathrm{r}}^2}{16\pi^2}\left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right)^{l+l'+1}\Gamma\left(\epsilon l + \epsilon l' + 2\epsilon\right) \\
&\quad \times \frac{1}{(-p^2)^{\epsilon(l+l'+2)}}\left[\frac{25}{12}g_{\mu\nu}p^2 - \frac{7}{3}p_\mu p_\nu - \frac{1}{4}\xi_{\mathrm{r}}(\delta_{0l}+\delta_{0l'})\left(g_{\mu\nu}p^2 - p_\mu p_\nu\right)\right] C_2(G)\delta^{ab}. \quad (46)
\end{aligned}
$$

The contribution of diagram $\mathrm{G_m}$ is given by

$$
\begin{aligned}
\Pi^{ab\ \mathrm{G_m}}_{\mu\nu}(p) &= -\frac{1}{2}\int \frac{d^n k}{i(2\pi)^n}(ig_{\mathrm{r}} f^{bcc'}) g_{\mathrm{r}}^3 N_f \\
&\quad \times \Bigg\{ \mathrm{Tr}(T^a T^d T^{d'})\mathrm{Tr}\left[\frac{1}{-\not k}\not p \frac{1}{-\not k}\not p \frac{1}{-(\not k + \not p)}\gamma_\mu \frac{1}{-\not k}K^{cdc'd'}_{\nu\ +}(k,p)\right] \\
&\quad + \mathrm{Tr}(T^d T^a T^{d'})\mathrm{Tr}\left[\frac{1}{-\not k}\gamma_\mu \frac{1}{-\not k}\not p \frac{1}{-\not k}\not p \frac{1}{-(\not k - \not p)}K^{cdc'd'}_{\nu\ -}(k,p)\right]\Bigg\}. \qquad (47)
\end{aligned}
$$

with the boson-fermion-fermion vertex parts

$$
K^{cdc'd'}_{\nu\ +}(k,p) = \int \frac{d^n q}{i(2\pi)^n}\gamma_\alpha \Xi_{\nu\beta\beta'}(q,p) D^{\alpha\beta\ cd\ [l]}(q) D^{\alpha'\beta'\ c'd'\ [l']}(p-q)\frac{1}{-(\not k + \not q)}\gamma_{\alpha'} \qquad (48)
$$

and

$$
K^{cdc'd'}_{\nu\ -}(k,p) = \int \frac{d^n q}{i(2\pi)^n}\gamma_{\alpha'} \Xi_{\nu\beta\beta'}(q,p) D^{\alpha\beta\ cd\ [l]}(q) D^{\alpha'\beta'\ c'd'\ [l']}(p-q)\frac{1}{-(\not k - \not q)}\gamma_{\alpha}. \qquad (49)
$$

Then the boson-fermion-fermion vertex parts $K^{cdc'd'}_{\nu\ +}(k,p)$ and $K^{cdc'd'}_{\nu\ -}(k,p)$ are calculated as

$$
\begin{aligned}
K^{cdc'd'}_{\nu\ +}(k,p) &= \frac{1}{16\pi^2}\left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right)^{l+l'}\Gamma(\epsilon l + \epsilon l' + \epsilon)\frac{\gamma_\nu}{(-k^2)^{\epsilon(l+l'+1)}}\left[-\frac{3}{2} - \frac{3}{4}\xi_{\mathrm{r}}(\delta_{0l}+\delta_{0l'})\right]\delta^{cd}\delta^{c'd'} \\
&\quad + O(m_{\mathrm{r}}^2) \qquad (50)
\end{aligned}
$$

and $K_{\nu\,-}^{cdc'd'}(k,p) = -K_{\nu\,+}^{cdc'd'}(k,p)$. Hence we obtain the contribution of $O(\epsilon^0)$ in diagram $\mathrm{G_m}$, as follows

$$\begin{aligned}\Pi_{\mu\nu}^{ab\ \mathrm{G_m}}(p) &= \frac{g_{\mathrm{r}}^2}{16\pi^2\Gamma(\epsilon)}\left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right)^{l+l'+1}\Gamma\left(\epsilon l+\epsilon l'+\epsilon\right)\Gamma\left(\epsilon l+\epsilon l'+2\epsilon\right)\\ &\quad\times\frac{1}{(-p^2)^{\epsilon(l+l'+2)}}\left[\frac{3}{4}+\frac{3}{8}\xi_{\mathrm{r}}(\delta_{0l}+\delta_{0l'})\right]\left(g_{\mu\nu}p^2-p_\mu p_\nu\right)C_2(G)\delta^{ab}. \qquad (51)\end{aligned}$$

For the diagrams $\mathrm{H_f}$ and $\mathrm{H_m}$, we can discuss the contributions to Z_3 in a similar manner to the diagrams $\mathrm{G_f}$ and $\mathrm{G_m}$. The contributions from diagrams $\mathrm{H_f}$ and $\mathrm{H_m}$ are calculated as

$$\begin{aligned}\Pi_{\mu\nu}^{ab\ \mathrm{H_f}}(p) &= \frac{g_{\mathrm{r}}^2}{16\pi^2}\left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right)^{l+l'+2}\Gamma\left(\epsilon l+\epsilon l'+3\epsilon\right)\\ &\quad\times\frac{1}{(-p^2)^{\epsilon(l+l'+3)}}\left[\frac{25}{12}g_{\mu\nu}p^2-\frac{7}{3}p_\mu p_\nu-\frac{1}{4}\xi_{\mathrm{r}}(\delta_{0l}+\delta_{0l'})\left(g_{\mu\nu}p^2-p_\mu p_\nu\right)\right]C_2(G)\delta^{ab} \qquad (52)\end{aligned}$$

and

$$\begin{aligned}\Pi_{\mu\nu}^{ab\ \mathrm{H_m}}(p) &= \frac{g_{\mathrm{r}}^2}{16\pi^2\Gamma(\epsilon)}\left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right)^{l+l'+2}\Gamma\left(\epsilon l+\epsilon l'+2\epsilon\right)\Gamma\left(\epsilon l+\epsilon l'+3\epsilon\right)\\ &\quad\times\frac{1}{(-p^2)^{\epsilon(l+l'+3)}}\left[-\frac{3}{4}-\frac{3}{8}\xi_{\mathrm{r}}(\delta_{0l}+\delta_{0l'})\right]\left(g_{\mu\nu}p^2-p_\mu p_\nu\right)C_2(G)\delta^{ab}, \qquad (53)\end{aligned}$$

respectively.

Next we discuss the total contributions from the diagrams in Fig.1. Hereafter we note *the number of one-fermion loops per the diagram* with the notation $\widetilde{l}$. Then the numbers $\widetilde{l}$ to the diagrams F, G, and H are given by $l+l'$, $l+l'+1$, and $l+l'+2$, respectively. For $\widetilde{l}=0$, the diagrams G and H are absent while the diagram F reduces to the diagram B.

First let us consider the contributions of *gauge independent part* to Z_3. The *gauge independent part* in the vacuum polarization tensor is noted with the subscript "0" on the tensor. In the total contribution from diagrams D and E, the *gauge independent part* does not contribute to Z_3 from (30) and (35). For $\widetilde{l}\geq 1$, *multiplicities* of diagrams F, $\mathrm{G_f}$, and $\mathrm{H_f}$ are $\widetilde{l}+1$, $2\widetilde{l}$, and $\widetilde{l}-1$, respectively, where the *multiplicities* is the numbers of diagrams with the same contribution to Z_3 for a number $\widetilde{l}$. In the total contribution from diagrams F, $\mathrm{G_f}$, and $\mathrm{H_f}$, when $\widetilde{l}\geq 1$, the *gauge independent parts* cancel each other, as follows

$$\begin{aligned}&(\widetilde{l}+1)\Pi_{\mu\nu\ 0}^{ab\ \mathrm{F}}(p)+2\widetilde{l}\Pi_{\mu\nu\ 0}^{ab\ \mathrm{G_f}}(p)+(\widetilde{l}-1)\Pi_{\mu\nu\ 0}^{ab\ \mathrm{H_f}}(p)\\ &=\frac{g_{\mathrm{r}}^2}{16\pi^2}\left[(\widetilde{l}+1)-2\widetilde{l}+(\widetilde{l}-1)\right]\left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right)^{\widetilde{l}}\frac{\Gamma(\epsilon\widetilde{l}+\epsilon)}{(-p^2)^{\epsilon(\widetilde{l}+1)}}\left(\frac{25}{12}g_{\mu\nu}p^2-\frac{7}{3}p_\mu p_\nu\right)C_2(G)\delta^{ab}\\ &=0 \qquad (54)\end{aligned}$$

from (38), (46), and (52). For $\widetilde{l}\geq 2$, the multiplicities of diagrams $\mathrm{G_m}$ and $\mathrm{H_m}$ are $2\widetilde{l}$ and $2(\widetilde{l}-1)$, respectively. In the total contribution from diagrams $\mathrm{G_m}$ and $\mathrm{H_m}$, when $\widetilde{l}\geq 2$, the *gauge independent part* is given by

$$2\widetilde{l}\Pi_{\mu\nu\ 0}^{ab\ \mathrm{G_m}}(p)+2(\widetilde{l}-1)\Pi_{\mu\nu\ 0}^{ab\ \mathrm{H_m}}(p)$$

$$
\begin{aligned}
&= \frac{g_\mathrm{r}^2}{16\pi^2\Gamma(\epsilon)}\left[2\widetilde{l} - 2(\widetilde{l}-1)\right]\left(-\frac{4}{3}N_f T(R) g_\mathrm{r}^2 I\right)^{\widetilde{l}} \frac{\Gamma(\epsilon\widetilde{l})\ \Gamma(\epsilon\widetilde{l}+\epsilon)}{(-p^2)^{\epsilon(\widetilde{l}+1)}}\left(\frac{3}{4}g_{\mu\nu}p^2 - \frac{3}{4}p_\mu p_\nu\right) C_2(G)\delta^{ab} \\
&\approx \frac{3}{2}\frac{g_\mathrm{r}^2 I}{\widetilde{l}(\widetilde{l}+1)}\left(-\frac{4}{3}N_f T(R) g_\mathrm{r}^2 I\right)^{\widetilde{l}} \left(g_{\mu\nu}p^2 - p_\mu p_\nu\right) C_2(G)\delta^{ab} \qquad (55)
\end{aligned}
$$

from (51) and (53). Hence the diagrams $\mathrm{G_m}$ and $\mathrm{H_m}$ with many one-fermion loops (i.e. $\widetilde{l} \geq 2$) contribute to Z_3 in total.

Now we consider the contributions of *gauge dependent part* to Z_3. The *gauge dependent part* in the vacuum polarization tensor is noted with the subscript "ξ" on the tensor. For $\widetilde{l} = 1$, the multiplicities of diagrams D and E are 2 and 1, respectively. From (30) and (35), the total contribution from diagrams D and E is given by

$$
\begin{aligned}
2\Pi_{\mu\nu}^{ab}{}_{\xi}^{\mathrm{D}}(p) + \Pi_{\mu\nu}^{ab}{}_{\xi}^{\mathrm{E}}(p) &= \xi_\mathrm{r}\frac{g_\mathrm{r}^2}{16\pi^2}\left(-\frac{4}{3}N_f T(R) g_\mathrm{r}^2 I\right)\frac{\Gamma(2\epsilon-1)}{(-p^2)^{2\epsilon}}\left(g_{\mu\nu}p^2 - p_\mu p_\nu\right) C_2(G)\delta^{ab} \\
&\approx -\xi_\mathrm{r}\frac{g_\mathrm{r}^2 I}{2}\left(-\frac{4}{3}N_f T(R) g_\mathrm{r}^2 I\right)\left(g_{\mu\nu}p^2 - p_\mu p_\nu\right) C_2(G)\delta^{ab}, \qquad (56)
\end{aligned}
$$

where we have used relation

$$
\mathrm{Tr}\left(T^a T^b T^c T^c\right) - \mathrm{Tr}\left(T^c T^a T^c T^b\right) = \frac{1}{4}C_2(G)\delta^{ab}. \qquad (57)
$$

Notice that the diagrams D and E with the many one-fermion loops does not contribute to Z_3. To the *gauge dependent part*, the diagrams F, G, and H have contributions only when one-fermion loop is not inserted on the gauge boson propagator, and then the factor $\delta_{0l} + \delta_{0l'}$ give 2 in (38), (46), (51), (52), and (53). For the *gauge dependent part* with $\widetilde{l} = 1$, we obtain the total contribution from diagrams D, E, F, $\mathrm{G_f}$, and $\mathrm{G_m}$ to Z_3,

$$
\begin{aligned}
&2\Pi_{\mu\nu}^{ab}{}_{\xi}^{\mathrm{D}}(p) + \Pi_{\mu\nu}^{ab}{}_{\xi}^{\mathrm{E}}(p) + \Pi_{\mu\nu}^{ab}{}_{\xi}^{\mathrm{F}}(p) + 2\Pi_{\mu\nu}^{ab}{}_{\xi}^{\mathrm{G_f}}(p) + 2\Pi_{\mu\nu}^{ab}{}_{\xi}^{\mathrm{G_m}}(p) \\
&\approx \xi_\mathrm{r} g_\mathrm{r}^2 I\left(-\frac{1}{2} - \frac{1}{4} + \frac{1}{2} + \frac{3}{4}\right)\left(-\frac{4}{3}N_f T(R) g_\mathrm{r}^2 I\right)\left(g_{\mu\nu}p^2 - p_\mu p_\nu\right) C_2(G)\delta^{ab} \\
&= \xi_\mathrm{r}\frac{g_\mathrm{r}^2 I}{2}\left(-\frac{4}{3}N_f T(R) g_\mathrm{r}^2 I\right)\left(g_{\mu\nu}p^2 - p_\mu p_\nu\right) C_2(G)\delta^{ab} \qquad (58)
\end{aligned}
$$

from (38), (46), (51), and (56). For $\widetilde{l} \geq 2$, the multiplicities of diagrams F, $\mathrm{G_f}$, and $\mathrm{H_f}$ are 1, 2, 1, respectively. In the total contribution from diagrams F, $\mathrm{G_f}$, and $\mathrm{H_f}$, when $\widetilde{l} \geq 2$, the *gauge dependent parts* cancel each other, as follows

$$
\begin{aligned}
&\Pi_{\mu\nu}^{ab}{}_{\xi}^{\mathrm{F}}(p) + 2\Pi_{\mu\nu}^{ab}{}_{\xi}^{\mathrm{G_f}}(p) + \Pi_{\mu\nu}^{ab}{}_{\xi}^{\mathrm{H_f}}(p) \\
&= \xi_\mathrm{r}\frac{(1-2+1)g_\mathrm{r}^2}{16\pi^2}\left(-\frac{4}{3}N_f T(R) g_\mathrm{r}^2 I\right)^{\widetilde{l}}\frac{\Gamma(\epsilon\widetilde{l}+\epsilon)}{(-p^2)^{\epsilon(\widetilde{l}+1)}}\left(-\frac{2}{4}g_{\mu\nu}p^2 + \frac{2}{4}p_\mu p_\nu\right) C_2(G)\delta^{ab} \\
&= 0 \qquad (59)
\end{aligned}
$$

from (38), (46), and (52). For $\widetilde{l} \geq 2$, the multiplicities of diagrams $\mathrm{G_m}$ and $\mathrm{H_m}$ are 2 and 2, respectively. In the total contribution from diagrams $\mathrm{G_m}$ and $\mathrm{H_m}$, when $\widetilde{l} \geq 2$, the *gauge dependent parts* cancel each other, as follows

$$\begin{aligned} & 2\Pi^{ab\ \mathrm{G_m}}_{\mu\nu\ \xi}(p) + 2\Pi^{ab\ \mathrm{H_m}}_{\mu\nu\ \xi}(p) \\ &= \xi_{\mathrm{r}} \frac{(2-2)g_{\mathrm{r}}^2}{16\pi^2\Gamma(\epsilon)} \left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right)^{\widetilde{l}} \frac{\Gamma(\epsilon\widetilde{l})\ \Gamma(\epsilon\widetilde{l}+\epsilon)}{(-p^2)^{\epsilon(\widetilde{l}+1)}} \left(-\frac{6}{8}g_{\mu\nu}p^2 + \frac{6}{8}p_\mu p_\nu\right) C_2(G)\delta^{ab} \\ &= 0 \end{aligned} \tag{60}$$

from (51) and (53).

3.2 Renormalization of subdiagram divergences

We discuss the renormalization to the subdiagram divergences. The divergent parts in diagrams are each one-fermion loop, the fermion self-energy part in D, the fermion-boson vertex part in E, the three-boson vertex part in $\mathrm{G_f}$ and $\mathrm{H_f}$, and the boson-fermion-fermion vertex part in $\mathrm{G_m}$ and $\mathrm{H_m}$. Hereafter we consider the minimal subtraction scheme.

For the divergence from one-fermion loop diagram, the counter term is given by

$$\Pi^{ab\ \mathrm{A}}_{\mu\nu\ \mathrm{c.t.}}(p) = -\frac{4}{3}\frac{1}{(4\pi)^2} N_f g_{\mathrm{r}}^2 \Gamma(\epsilon)\left(-g_{\mu\nu}p^2 + p_\mu p_\nu\right) \mathrm{Tr}(T^a T^b) \tag{61}$$

from (14). Then the gauge boson propagator (25) with many one-fermion loops is replaced by renormalized one, as follows

$$\begin{aligned} D^{ab\ [l]}_{\mu\nu\ \mathrm{ren}}(p) &= \left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right)^l \left[\frac{1}{(-p^2)^\epsilon} - 1\right]^l \frac{1}{(-p^2)} \left[-g_{\mu\nu} + (1-\xi_{\mathrm{r}}\delta_{0l})\frac{p_\mu p_\nu}{p^2}\right]\delta^{ab} \\ &= \left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right)^l \sum_{n=0}^{l} \frac{l!}{n!(l-n)!} \frac{(-1)^{l-n}}{(-p^2)^{1+\epsilon n}} \left[-g_{\mu\nu} + (1-\xi_{\mathrm{r}}\delta_{0l})\frac{p_\mu p_\nu}{p^2}\right]\delta^{ab}, \end{aligned} \tag{62}$$

where n is the number of one-fermion loops in the gauge boson propagator, and $l-n$ is the number of counter terms in the gauge boson propagator for the divergences from one-fermion loops.

To the divergences of fermion self-energy part (29) and vertex correction part (34), the counter terms are given by

$$J_{\mathrm{c.t.}}(k) = \xi_{\mathrm{r}} \frac{g_{\mathrm{r}}^2}{16\pi^2}\Gamma(\epsilon)\ \not{k} T^c T^d \delta^{cd} \tag{63}$$

and

$$\Lambda^a_{\mu\ \mathrm{c.t.}} = -\xi_{\mathrm{r}} \frac{2g_{\mathrm{r}}^3}{16\pi^2}\Gamma(\epsilon)\gamma_\mu T^d T^a T^c \delta^{cd}, \tag{64}$$

respectively. Then (30), (35), and (56) are replaced by

$$\Pi^{ab\ \mathrm{D}}_{\mu\nu\ \mathrm{ren}}(p) = \xi_{\mathrm{r}} \frac{g_{\mathrm{r}}^2}{8\pi^2}\left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right)\left[\frac{\Gamma(2\epsilon-1)}{(-p^2)^{2\epsilon}} - \frac{\Gamma(\epsilon-1)}{(-p^2)^\epsilon}\right]\left(g_{\mu\nu}p^2 - p_\mu p_\nu\right)\mathrm{Tr}\left(T^a T^b T^c T^d\right)\delta^{cd}, \tag{65}$$

$$\Pi_{\mu\nu\ \mathrm{ren}}^{ab\ \mathrm{E}}(p) = -\xi_{\mathrm{r}} \frac{2g_{\mathrm{r}}^2}{8\pi^2} \left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right) \left[\frac{\Gamma(2\epsilon-1)}{(-p^2)^{2\epsilon}} - \frac{\Gamma(\epsilon-1)}{(-p^2)^{\epsilon}}\right] \left(g_{\mu\nu}p^2 - p_\mu p_\nu\right) \mathrm{Tr}\left(T^d T^a T^c T^b\right) \delta^{cd}, \quad (66)$$

and

$$\begin{aligned} &2\Pi_{\mu\nu\ \xi\ \mathrm{ren}}^{ab\ \mathrm{D}}(p) + \Pi_{\mu\nu\ \xi\ \mathrm{ren}}^{ab\ \mathrm{E}}(p) \\ &= \xi_{\mathrm{r}} \frac{g_{\mathrm{r}}^2}{16\pi^2} \left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right) \left[\frac{\Gamma(2\epsilon-1)}{(-p^2)^{2\epsilon}} - \frac{\Gamma(\epsilon-1)}{(-p^2)^{\epsilon}}\right] \left(g_{\mu\nu}p^2 - p_\mu p_\nu\right) C_2(G)\delta^{ab} \\ &\approx \xi_{\mathrm{r}} \frac{g_{\mathrm{r}}^2 I}{2} \left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right) \left(g_{\mu\nu}p^2 - p_\mu p_\nu\right) C_2(G)\delta^{ab}, \qquad (67) \end{aligned}$$

respectively.

We take into account the counter terms for the divergence from many one-fermion loops in the diagrams F, G, and H. Then (38) is replaced by

$$\begin{aligned} \Pi_{\mu\nu\ \mathrm{ren}}^{ab\ \mathrm{F}}(p) &= \frac{g_{\mathrm{r}}^2}{16\pi^2} \left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right)^{\widetilde{l}} \sum_{\widetilde{n}=0}^{\widetilde{l}} \frac{\widetilde{l}!}{\widetilde{n}!(\widetilde{l}-\widetilde{n})!} \Gamma\left(\epsilon\widetilde{n}+\epsilon\right) \frac{(-1)^{\widetilde{l}-\widetilde{n}}}{(-p^2)^{\epsilon(\widetilde{n}+1)}} \\ &\times \left[\frac{25}{12}g_{\mu\nu}p^2 - \frac{7}{3}p_\mu p_\nu - \frac{1}{4}\xi_{\mathrm{r}}(\delta_{0l}+\delta_{0l'})\left(g_{\mu\nu}p^2 - p_\mu p_\nu\right)\right] C_2(G)\delta^{ab} \qquad (68) \end{aligned}$$

with $\widetilde{l} = l + l'$ and $\widetilde{n} = n + n'$, where n and n' are numbers of one fermion loops in each gauge boson propagator, and $l-n$ and $l'-n'$ are numbers of counter terms for the divergence from one-fermion loops in each gauge boson propagator. To the divergence from three boson vertex part (45), the counter term is given by

$$F_{\mu\alpha\alpha'\ \mathrm{c.t.}}^{add'}(q,p) = -ig_{\mathrm{r}} f^{add'} \left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right) \left[g_{\mu\alpha}(p+q)_{\alpha'} + g_{\mu\alpha'}(q-2p)_\alpha + g_{\alpha\alpha'}(p-2q)_\mu\right]. \qquad (69)$$

Then we replace (46) and (51) by

$$\begin{aligned} \Pi_{\mu\nu\ \mathrm{ren}}^{ab\ \mathrm{G_f}}(p) &= \frac{g_{\mathrm{r}}^2}{16\pi^2} \left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right)^{\widetilde{l}} \sum_{\widetilde{n}=0}^{\widetilde{l}-1} \frac{(\widetilde{l}-1)!}{\widetilde{n}!(\widetilde{l}-\widetilde{n}-1)!} \left[\frac{1}{(-p^2)^{\epsilon}}\Gamma(\epsilon\widetilde{n}+2\epsilon) - \Gamma(\epsilon\widetilde{n}+\epsilon)\right] \\ &\times \frac{(-1)^{\widetilde{l}-\widetilde{n}}}{(-p^2)^{\epsilon(\widetilde{n}+1)}} \left[\frac{25}{12}g_{\mu\nu}p^2 - \frac{7}{3}p_\mu p_\nu - \frac{1}{4}\xi_{\mathrm{r}}(\delta_{0l}+\delta_{0l'})\left(g_{\mu\nu}p^2 - p_\mu p_\nu\right)\right] C_2(G)\delta^{ab} \quad (70) \end{aligned}$$

and

$$\begin{aligned} \Pi_{\mu\nu\ \mathrm{ren}}^{ab\ \mathrm{G_m}}(p) &= \frac{g_{\mathrm{r}}^2}{16\pi^2} \left(-\frac{4}{3}N_f T(R) g_{\mathrm{r}}^2 I\right)^{\widetilde{l}} \sum_{\widetilde{n}=0}^{\widetilde{l}-1} \frac{(\widetilde{l}-1)!}{\widetilde{n}!(\widetilde{l}-\widetilde{n}-1)!} \Gamma(\epsilon\widetilde{n}+\epsilon) \left[\frac{1}{(-p^2)^{\epsilon\widetilde{n}}}\frac{\Gamma(\epsilon\widetilde{n}+2\epsilon)}{\Gamma(\epsilon)} - 1\right] \\ &\times \frac{(-1)^{\widetilde{l}-\widetilde{n}}}{(-p^2)^{\epsilon}} \left[-\frac{3}{4} - \frac{3}{8}\xi_{\mathrm{r}}(\delta_{0l}+\delta_{0l'})\right] \left(g_{\mu\nu}p^2 - p_\mu p_\nu\right) C_2(G)\delta^{ab}, \qquad (71) \end{aligned}$$

respectively, where $\widetilde{l}=l+l'+1$ and $\widetilde{n}=n+n'$. In the similar manner, (52) and (53) are replaced by

$$\Pi^{ab\ \mathrm{H_f}}_{\mu\nu\ \mathrm{ren}}(p) = -\Pi^{ab\ \mathrm{G_f}}_{\mu\nu\ \mathrm{ren}}(p) \tag{72}$$

$$\Pi^{ab\ \mathrm{H_m}}_{\mu\nu\ \mathrm{ren}}(p) = -\Pi^{ab\ \mathrm{G_m}}_{\mu\nu\ \mathrm{ren}}(p) \tag{73}$$

with $\widetilde{l}=l+l'+2$ and $\widetilde{n}=n+n'$.

The total contribution (55) is replaced by

$$\begin{aligned}
&2\widetilde{l}\Pi^{ab\ \mathrm{G_m}}_{\mu\nu\ 0\ \mathrm{ren}}(p) + 2(\widetilde{l}-1)\Pi^{ab\ \mathrm{H_m}}_{\mu\nu\ 0\ \mathrm{ren}}(p) \\
&\approx \left[2\widetilde{l}-2(\widetilde{l}-1)\right]\frac{g_\mathrm{r}^2}{16\pi^2\epsilon}\left(-\frac{4}{3}N_fT(R)g_\mathrm{r}^2I\right)^{\widetilde{l}}\sum_{\widetilde{n}=0}^{\widetilde{l}-1}\frac{(\widetilde{l}-1)!}{\widetilde{n}!(\widetilde{l}-\widetilde{n}-1)!}\frac{1}{(\widetilde{n}+1)}\left(\frac{1}{\widetilde{n}+2}-1\right)(-1)^{\widetilde{l}-\widetilde{n}} \\
&\times\left(-\frac{3}{4}g_{\mu\nu}p^2+\frac{3}{4}p_\mu p_\nu\right)C_2(G)\delta^{ab} \\
&= \frac{3}{2}\frac{g_\mathrm{r}^2I}{\widetilde{l}(\widetilde{l}+1)}\left(\frac{4}{3}N_fT(R)g_\mathrm{r}^2I\right)^{\widetilde{l}}\left(g_{\mu\nu}p^2-p_\mu p_\nu\right)C_2(G)\delta^{ab}
\end{aligned} \tag{74}$$

from (71) and (73). The total contribution (58) is replaced by

$$\begin{aligned}
&2\Pi^{ab\ \mathrm{D}}_{\mu\nu\ \xi\ \mathrm{ren}}(p) + \Pi^{ab\ \mathrm{E}}_{\mu\nu\ \xi\ \mathrm{ren}}(p) + \Pi^{ab\ \mathrm{F}}_{\mu\nu\ \xi\ \mathrm{ren}}(p) + 2\Pi^{ab\ \mathrm{G_f}}_{\mu\nu\ \xi\ \mathrm{ren}}(p) + 2\Pi^{ab\ \mathrm{G_m}}_{\mu\nu\ \xi\ \mathrm{ren}}(p) \\
&\approx \xi_\mathrm{r}\frac{g_\mathrm{r}^2I}{2}\left(\frac{4}{3}N_fT(R)g_\mathrm{r}^2I\right)\left(g_{\mu\nu}p^2-p_\mu p_\nu\right)C_2(G)\delta^{ab}
\end{aligned} \tag{75}$$

from (67), (68), (70), and (71).

Hence we obtain the total contribution from all diagrams in Fig.1, as follows

$$\begin{aligned}
\Pi^{ab}_{\mu\nu\ \mathrm{div}} &= \left[-\frac{4}{3}g_\mathrm{r}^2IN_fT(R)+\frac{2}{3}\xi_\mathrm{r}(g_\mathrm{r}^2I)^2N_fT(R)C_2(G)+\left(\frac{13}{6}-\frac{1}{2}\xi_\mathrm{r}\right)g_\mathrm{r}^2IC_2(G)\right](g_{\mu\nu}p^2-p_\mu p_\nu)\delta^{ab} \\
&\quad+\sum_{\widetilde{l}=1}^{\infty}\frac{2}{\widetilde{l}(\widetilde{l}+1)}\left(\frac{4}{3}\right)^{\widetilde{l}-1}(g_\mathrm{r}^2I)^{\widetilde{l}+1}\left[N_fT(R)\right]^{\widetilde{l}}(g_{\mu\nu}p^2-p_\mu p_\nu)C_2(G)\delta^{ab}
\end{aligned} \tag{76}$$

from (74) and (75). Then the renormalization constant Z_3 is given by

$$\begin{aligned}
Z_3 &= 1-\frac{4}{3}g_\mathrm{r}^2IN_fT(R)+\sum_{\widetilde{l}=1}^{\infty}\frac{2}{\widetilde{l}(\widetilde{l}+1)}\left(\frac{4}{3}\right)^{\widetilde{l}-1}(g_\mathrm{r}^2I)^{\widetilde{l}+1}\left[N_fT(R)\right]^{\widetilde{l}}C_2(G) \\
&\quad+\frac{2}{3}\xi_\mathrm{r}(g_\mathrm{r}^2I)^2N_fT(R)C_2(G)+g_\mathrm{r}^2I\left(\frac{13}{6}-\frac{1}{2}\xi_\mathrm{r}\right)C_2(G).
\end{aligned} \tag{77}$$

Therefore we obtain the renormalization constant Z_3 to non-abelian gauge field at the leading and the next-to-leading order in N_f^{-1}, as follows

$$Z_3 = 1-\frac{4}{3}g_\mathrm{r}^2IN_fT(R)+\frac{11}{3}g_\mathrm{r}^2IC_2(G)-\frac{1}{2}\xi_\mathrm{r}g_\mathrm{r}^2IC_2(G)\left[1-\frac{4}{3}g_\mathrm{r}^2IN_fT(R)\right]$$

$$+\frac{3}{2}g_{\rm r}^2 IC_2(G)\left[\frac{4}{2g_{\rm r}^2 IN_f T(R)}-1\right]\log_e\left[1-\frac{4}{3}g_{\rm r}^2 IN_f T(R)\right] \tag{78}$$

with $I \approx 1/(16\pi^2\epsilon)$. In (78), the third, foruth, and fifth terms arise from the next-to-leading contribution in the $1/N_f$ expansion.

4 SOLVING THE COMPOSITENESS CONDITION

We have discussed so far the renormalization constant Z_3 in the general non-abelian gauge field theory (7) at the next-to-leading order in $1/N_f$. The information of composite gauge field theory (1) can be obtained from the gauge field theory with $Z_3 = 0$. Now we use the compositeness condition on (78) and solve it for $g_{\rm r}$. The equation (20) shows that

$$g_{\rm r}^2 = \frac{3}{4N_f T(R) I} + O\left(\frac{1}{N_f^2}\right). \tag{79}$$

Substituting (79) to the equation $Z_3 = 0$ with (78), we see that the logarithmic term and the gauge dependent term in (78) are the order of $1/N_f^2$, and then these terms are negligible in the present approximation. Hence $Z_3 = 0$ reduces to the simple equation

$$1-\frac{4}{3}g_{\rm r}^2 IN_f T(R)+\frac{11}{3}g_{\rm r}^2 IC_2(G)=0. \tag{80}$$

Therefore we obtain

$$g_{\rm r}^2 = \frac{3}{\left[4N_f T(R) - 11C_2(G)\right] I} \tag{81}$$

at the next-to-leading order in $1/N_f$ expansion. In (81), if the following relation holds,

$$N_f > \frac{11C_2(G)}{4T(R)}, \tag{82}$$

$g_{\rm r}^2$ has positive value, and then the gauge boson may be a composite of the matter fermion and its anti-particle. In the general non-abelian gauge field theory, the marginal value for asymptotic freedom is given by $N_f T(R) = 11C_2(G)/4$. Hence we find that the gauge boson can be composite when the fermion-flavors number N_f satisfy the condition (82), in which the gauge field theory is not asymptotically free. Therefore we find that the allowed region of N_f in (82) for the gauge boson compositeness is *complementary* to the asymptotic freedom in the gauge field theory.

When these results are applied to $SU(2)_L$ of the Glashow-Weinberg-Salam (GWS) theory with twelve flavors $N_f = 12$ (three families of leptons and three families with three colors of quarks) and $SU(2)$ colors ($C_2(G) = N_c = 2$) of their matter fermions, (82) holds, and then the weak bosons can be a composite of the matter fermion and its anti-particle. If we apply the model to the quantum chromodynamics (QCD) with six flavors $N_f = 6$ (three families of quarks) and $SU(3)$ colors ($C_2(G) = N_c = 3$) of quarks, then the gluons can not be a composite of the quark and its anti-particle, because (82) does not hold. In

an application of the model to hadron physics, we can discuss the compositeness of light mesons. From (82), the ρ meson does not become composite gauge boson within this model, because the flavors number is $N_f = 2$ (one family of light quark) though the $SU(3)$ colors is given by $C_2(G) = N_c = 3$. In an application of the model to the system with matter fermions which belong to the adjoint representation of $SU(N_c)$ color gauge group, the gauge bosons can be a composite of the fermion and its anti-particle when the fermion-flavors number satisfies $N_f \geq 3$ from (82). In this case, the allowed region of N_f does not depend on the colors number N_c.

In the abelian gauge theory, the gauge boson can be always a composite of the matter fermions, because the next-to-leading order contribution suppressed in the theory as shown in the previous paper [12]. If we apply the results to quantum electrodynamics, the photon may be always a composite of $U(1)$ charged matter fermion and its anti-particle.

In $1/N_f$ expansion, the flavors number N_f need to has the large value in order to obtain the appropriate approximation. In this model, when (82) holds, the coupling constant (81) is rewritten as

$$g_{\mathrm{r}}^2 = \frac{3}{4N_f T(R) I} \left[1 + \frac{11 C_2(G)}{4N_f T(R)} \right], \tag{83}$$

in which the second term in bracket gives the next-to-leading contribution. For the appropriate approximation, the next-to-leading contribution in (83) need to becomes the small. For example, we apply (83) to the system with matter fermions ($N_f = 12$) which belong to the adjoint representation of $SU(2)$ color gauge group. Then the second term in bracket of (83) give the following value,

$$\frac{11 C_2(G)}{4N_f T(R)} = \frac{11}{4N_f} \approx 0.23, \tag{84}$$

in which the next-to-leading contribution is sufficiently small. In this system, let us estimate the order of coupling constant of composite gauge field theory from (83). If we fix the scales as $\Lambda = 10^4$ GeV and $m_{\mathrm{r}} = 10^{-3}$ GeV, then (83) give

$$\alpha_{\mathrm{r}} = \frac{g_{\mathrm{r}}^2}{4\pi} \approx 0.03, \tag{85}$$

in which the order of α_{r} is equal to the order of $SU(2)$ weak coupling constant in the GWS theory.

5 CONCLUSIONS

In order to investigate the composite gauge field, we have considered the compositeness condition in the general non-abelian gauge theory with fermionic matter fields. The gauge field theory with compositeness condition $Z_3 = 0$ is equivalent to the composite gauge field theory where Z_3 is the wave function renormalization constant of the gauge field. At the leading order in $1/N_f$ expansion, the gauge coupling constant g_{r} is proportional to $\sqrt{\epsilon/N_f}$ where N_f is the number of matter-fermion flavors and ϵ is related to the cutoff scale Λ. In the general non-abelian gauge field theory, we have calculated the renormalization constant Z_3 at the next-to-leading order in $1/N_f$ expansion. Through the compositeness condition ($Z_3 = 0$), we obtain the coupling constant in composite gauge field theory as $g_{\mathrm{r}} \propto 1/\sqrt{4N_f T(R) - 11 C_2(G)}$. Then we have found that the gauge boson can be a composite of the matter fermion and its anti-particle when N_f

satisfies the relation $N_f > 11C_2(G)/4T(R)$ because $g_{\rm r}$ has to be real, in which the non-abelian gauge field theory is not asymptotically free. Therefore we find *the complementarity* that the allowed region of N_f for the gauge boson compositeness is complementary to the asymptotic freedom in the non-abelian gauge field theory. We have applied these results to the weak bosons in GWS theory and the gluons in QCD, and the gauge bosons in the system with matter fermions which belong to the adjoint representation of $SU(N_c)$ color gauge group. Then the weak bosons can be a composite of the quarks and the leptons, but the gluons can not be a composite of the quarks. The gauge bosons can be a composite of fermions which belong to the adjoint representation of $SU(N_c)$ color gauge group if the fermion-flavors number satisfies $N_f \geq 3$. To the application of compositeness condition in abelian gauge theory, the photon in quantum electrodynamics may be always a composite of the $U(1)$ charged matter fermions, because the next-to-leading order contribution suppressed within the abelian gauge theory as shown in the previous paper [12]. We have seen that when the relation $N_f > 11C_2(G)/4T(R)$ holds, the $1/N_f$ expansion gives the appropriate approximation. Then we can estimate the order of coupling constant of composite gauge field theory.

ACKNOWLEDGMENTS

We would like to thank Professor K. Akama for helpful discussions. We would like to thank Professor H. Terazawa for kind hospitality during our stay at KEK.

References

[1] W. Heisenberg, Rev. Mod. Phys. **29**, 269 (1957); J. D. Bjorken, Ann. Phys. **24**, 174 (1963).

[2] K. Akama and H. Terazawa, INS-Rep-**257** (1976); H. Terazawa, Y. Chikashige and K. Akama, Phys. Rev. **D15**, 480(1977); Prog. Theor. Phys. **56**, 1935 (1976) ; T. Saito and K. Shigemoto, Prog. Theor. Phys. **57** (1977) 242; O. W. Greenberg and J. Sucher, Phys. Lett. **99B**, 339 (1981); H. Terazawa, Phys. Rev. **D22**, 184 (1980); Prog. Theor. Phys. **64**, 1763 (1980);**79**, 734 (1988); Mod. Phys. Lett. **A13**, 2427 (1988); L. F. Abbott and E. Farhi, Phys. Lett. **101B**, 69 (1981). M. Yasuè, Nucl. Phys. **B234**, 252 (1983); Prog. Theor. Phys. **81**, 271 (1989); Phys. Rev. **D39**, 3458 (1989);**D42**, 3169 (1990);

[3] See for example, J. J. Sakurai, *Currents and Mesons*, (Univ. Chicago Press, Chicago, 1969); M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, Phys. Rev. Lett. **54** 1215 (1985).

[4] A. Hasenfratz and P. Hasenfratz, Phys. Lett. **B297**, 166 (1992); M. A. Luty, Phys. Rev. **D48**, 1295 (1993); M. Bando, Y. Taniguchi and S. Tanimura, Prog. Theor. Phys. **97**, 665 (1997); M. Bando, J. Sato, and K. Yoshioka, ICRR-Report-384-97-7, hep-ph/9703321 (1997); M. Bando, hep-ph/9705237 (1997).

[5] K. Akama and T. Hattori, Phys. Rev. **D40**, 3688 (1989);**D51**, 3895 (1995); Int. J. Mod. Phys. **A9**, 3503 (1994); K. Akama, T. Hattori, and M. Yasuè, Phys. Rev. **D42**, 789 (1990);**D43**, 1702 (1991); B. S. Balakrishna and K. T. Mahanthappa, Phys. Rev. **D49** 2653 (1994); Phys. Rev. **D52** 2379 (1995); A. Galli, Phys. Rev. **D51**, 3876 (1995); Nucl. Phys. **B435**, 339 (1995); C. P. Burgess, J.-P. Derendinger, F. Quevedo, and M. Quiros, Phys. Lett. **B348** 428 (1995); V. V. Kabachenko and Y. F. Pirogov, IHEP 96-97, hep-ph/9612275 (1996); M. Berkooz, P. Cho, P. Kraus, and M. J. Strassler, HUTP-97/A014, hep-th/9705003 (1997); M. Yasuè, TOKAI-HEP/TH-9701, hep-ph/9707312 (1997).

[6] B. Jouvet, Nuovo Cim. **5**, 1133 (1956); M. T. Vaughn, R. Aaron and R. D. Amado, Phys. Rev. **124**, 1258 (1961); A. Salam, Nuovo Cim. **25**, 224 (1962); S. Weinberg, Phys. Rev. **130**, 776 (1963); I. Bialynicki-Birula, Phys. Rev. **130**, 465 (1963); D. Lurié and A. J. Macfarlane, Phys. Rev. **136** (1964) B816; T. Eguchi, Phys. Rev. **D14** (1976) 2755; **D17** (1978) 611; K. Shizuya, Phys. Rev. **D21** (1980) 2327.

[7] D. J. Gross and A. Neveu, Phys. Rev. **D10**, 3235 (1974); T. Kugo, Prog. Theor. Phys. **55**, 2032 (1976); T. Kikkawa, Prog. Theor. Phys. **56**, 947 (1976).

[8] M. Tanabashi, Phys. Lett. **B384**, 218 (1996); M. C. Birse, Z. Phys. **A355**, 231 (1996); M. Hashimoto, Phys. Rev. **D54**, 5619 (1996); C. M. Shakin and W.-D. Sun, Phys. Rev. **D55**, 2874 (1997); L.-H. Chan, Phys. Rev. **D55**, 5362 (1997); M. Harada and A. Shibata, Phys. Rev. **D55**, 6716 (1997).

[9] K. Akama and H. Terazawa, Phys. Lett. **112B**, 387 (1982);**321B**, 145 (1994); Phys. Rev. D **55**, R2521 (1997); M. Bander, Phys. Rev. Lett. 77, 601 (1996); A. E. Nelson, Phys. Rev. Lett. **78** 4159 (1997); S. L. Adler, IASSNS-HEP 97-12, hep-ph/9702378; M. C. Gonzalez-Garcia and S.F. Novaes, IFT-P-024-97, hep-ph/9703346; N. Di Bartolomeo and M. Fabbrichesi, Phys. Lett. **B406**, 237 (1997); K. Akama K. Katsuura, and H. Terazawa, Phys. Rev. D **56**, R2490 (1997).

[10] W.A. Bardeen, C.T. Hill and M. Lindner, Phys. Rev. **D41**, 1647 (1990); M. Harada, Y. Kikukawa, T. Kugo and H. Nakano, Prog. Theor. Phys. **92**, 1161 (1994).

[11] K. Akama, in Gauge Theory and Gravitation, Proceedings of the International Symposium, Nara, Japan, 1982 ed. K. Kikkawa, N. Nakanishi and H. Nariai (Springer-Verlag) p. 267; M. D. Maia, J. Math. Phys. **25** (1984) 2090; K. Akama, Prog. Theor. Phys. **78**, 184 (1987); **79**, 1299 (1988).

[12] K. Akama and T. Hattori, Phys. Lett. **B392**, 383 (1997); Phys. Lett. **B445**, 106 (1998);

[13] CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **77**, 438 (1996); *ibid.* **77**, 5336 (1996); Phys. Rev. D **55**, R5263 (1997); Particle Data Group, R.M. Barnett *et al.*, Phys. Rev. D **54**, 1(1996); H1 Collaboration, C. Adloff *et al.*, Z. Phys. C. **74**, 191 (1997); ZEUS Collaboration, J. Breitweg *et al.*, Z. Phys. C. **74**, 207 (1997).

[14] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).

[15] K. Akama, Phys. Rev. Lett. **76**, 184 (1996).

[16] D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973); H. D. Politzer, Phys. Rev. Lett. **30**, 1346 (1973).

[17] See for example, T. Muta, *Foundations of Quantum Chromodynamics, An Introduction to Perturbative Method in Gauge Theories*, World Scientific, (1987).

Figure 1: The gauge boson self-energy parts at the leading order (A) and at the next-to-leading order (B-H) in $1/N_f$. The solid, wavy, and dotted lines indicate the fermion, gauge boson, and Fadeev-Popov ghost propagators, respectively. The disk stands for insertion of an arbitrary number of one-fermion-loops into the gauge boson propagator. p indicates the momentum of boson in the external line. q and k are momenta in the internal line. a and b are color indices.

Figure 2: The separated overlapping divergence. $\mathrm{G_f}$, $\mathrm{H_f}$: the part where the divergence occurs at the fermion loop subdiagram in the three boson vertex part. $\mathrm{G_m}$, $\mathrm{H_m}$: the part where the divergence occurs at the boson-fermion (mixed) loop subdiagram in the boson-fermion vertex part.